LMU-TPW 007
UAHEP002
hep-th/0002170

# Four-point Functions of Lowest Weight CPOs in $\mathcal{N}=4$ SYM$_4$ in Supergravity Approximation

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**Abstract**

We show that the recently found quartic action for the scalars from the massless graviton multiplet of type IIB supergravity compactified on $AdS_5 \times S^5$ background coincides with the relevant part of the action of the gauged $\mathcal{N}=8$ 5d supergravity on $AdS_5$. We then use this action to compute the 4-point function of the lowest weight chiral primary operators $\mathrm{tr}\,(\phi^{(i}\phi^{j)})$ in $\mathcal{N}=4$ SYM$_4$ at large $N$ and at strong ‘t Hooft coupling.

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# 1 Introduction

The AdS/CFT duality [1, 2, 3] provides a remarkable way to approach the problem of studying correlation functions in certain conformal field theories. For $\mathcal{N} = 4$ supersymmetric Yang-Mills theory in four dimensions (SYM$_4$) this duality allows one to find the generating functional of Green functions of some composite gauge invariant operators at large $N$ and at strong 't Hooft coupling $\lambda$ by computing the on-shell value of the type IIB supergravity action on $AdS_5 \times S^5$ background [2, 3].

Thus, the knowledge of type IIB supergravity action up to n-th order in perturbation of fields near their background values is a necessary starting point for computing $n$-point correlation functions of corresponding operators in SYM$_4$. At present the quadratic [4] and cubic [5, 6, 7] actions for physical fields of type IIB supergravity are available that allows one to determine normalizations for many two- and three-point functions.

With four-point functions the situation is much more involved [8]-[24]. So far the only known examples here are the 4-point functions of operators $\mathrm{tr}\,(\,F^2 + \cdots)$ and $\mathrm{tr}\,(\,F\tilde{F} + \cdots)$ [8, 16] that on the gravity side correspond to massless modes of dilaton and axion fields, where the relevant part of the gravity action was known. These operators are rather complicated, in particular, in representation of the supersymmetry algebra they appear as descendents of the primary operators $O_2^I = \mathrm{tr}\,(\,\phi^{(i}\phi^{j)})$, where $\phi^i$ are Yang-Mills scalars transforming in the fundamental representation of the $R$-symmetry group $SO(6)$. The descendent nature of these operators brings considerable complications both in perturbative analysis of the correlation functions, and in study of their Operator Product Expansion (OPE) from AdS gravity [21].

More generally in $\mathcal{N} = 4$ SYM$_4$ there are chiral multiplets generated by (single-trace) chiral primary operators (CPO): $O_k^I = \mathrm{tr}\,(\,\phi^{(i_1} \cdots \phi^{i_k)})$, transforming in the $k$-traceless symmetric representation of $SO(6)$. Eight from sixteen supercharges annihilate $O_k^I$ while the other eight generate, under supersymmetry transformations, the chiral multiplets. A fundamental property of CPOs is that they have conformal dimensions protected against quantum corrections. Thus, they may be viewed as BPS states preserving 1/2 of the supersymmetry. In particular, the lowest component CPOs $O_2^I$ comprise together with their descendents a multiplet containing the stress-energy tensor and the $R$-symmetry current.

Recently we have found the quartic effective 5d action for scalar fields $s^I$ that correspond at linear order to chiral primary operators $O^I$ [25]. We have also shown that the found action admits a consistent Kaluza-Klein (KK) truncation to fields from the massless graviton multiplet. This multiplet represents a field content of the gauged $\mathcal{N} = 8$, $d = 5$ supergravity [26, 27, 28] and by the AdS/CFT correspondence it is dual to the Yang-Mills stress-energy multiplet.

Clearly, these results provide a possibility to find four-point functions of *any* CPOs[1] in supergravity approximation. In this paper as the first step in this direction we compute the simplest four-point correlation functions for all lowest weight CPOs $O_2^I$. Hopefully, this will further extend our understanding of the OPE in $\mathcal{N}=4$ SYM$_4$ at strong coupling. The detailed study of the OPE of two lowest weight CPOs will be the subject of a separate paper.

We start by showing that the quartic action [25] found by compactifying IIB supergravity on the $AdS_5 \times S^5$ with the further reduction to the massless multiplet coincides after some additional field redefinitions with relevant part of the action for the gauged $\mathcal{N}=8$ five-dimensional supergravity on $AdS_5$. This fact together with consistency of the KK reduction demonstrates, in particular, that within the supergravity approach, four-point correlation functions for fields from the YM stress-energy multiplet are completely determined by the 5d gauged supergravity, i.e., they do not receive any contributions from higher KK modes.

The gauged $\mathcal{N}=8$ five-dimensional supergravity has 42 scalars with 20 of them forming a singlet of the global invariance group $SL(2,\mathbf{R})$. These 20 scalars $s^I$ comprise the **20** irrep. of $SO(6)$ and correspond to CPOs $O^I = C^I_{ij}\,\mathrm{tr}\,(\phi^i\phi^j)$, where $C^I_{ij}$ is a traceless symmetric tensor of $SO(6)$. As we will see the only fields that appear in Feynman exchange diagrams describing the contribution to the 4-point function of $O^I$ are the scalars $s^I$, the graviton and the massless vector fields. There are also contributions of contact diagrams corresponding to quartic couplings of $s^I$ with two-derivatives and without derivatives.

The paper is organized as follows. In Section 2 we summarize the results of the KK reduction obtained in [25] and put the action in a form suitable for comparison with the action of gauged 5d supergravity. In Section 3 we employ an explicit parametrization for the coset space $SL(6,\mathbf{R})/SO(6)$ to write down the relevant part of the action for gauged 5d supergravity. We then decompose this action near $AdS_5$ background solution and after an additional field redefinition find an exact agreement with the action obtained by the KK reduction. Finally in Section 3 we combine our knowledge of the action with the technique [16] of computing exchange Feynman diagrams over the AdS space and give an answer for the 4-point function of lowest weight CPOs in terms of universal $D$-functions. Some technical details are relegated to two Appendices.

[1] The fields $s^I$ correspond to extended CPOs involving single- and multi-trace CPOs and their descendents, see [6, 25]. However, for generic values of conformal dimensions CPOs and extended CPOs have the same correlation functions.

## 2 Results of the reduction

As was discussed in the Introduction, the computation of a four-point function of arbitrary CPOs requires the construction of the effective 5d gravity action with all cubic terms involving two fields $s^I$ and with all $s^I$-dependent quartic terms, the problem that has been completely solved in [25]. For the simplest case of lowest weight CPOs the corresponding gravity fields are 20 scalars $s^I$ with the lowest AdS-mass $m^2 = -4$ and they are in the massless graviton multiplet. If we restrict our attention to these fields $s^I$ then the relevant part of the action may be written in the form [25]:

$$S(s) = \frac{4N^2}{(2\pi)^5}\int d^5x\sqrt{-g_a} \quad \Big( \mathcal{L}_2(s) + \mathcal{L}_2(\varphi_{\mu\nu}) + \mathcal{L}_2(A_\mu) \\ + \mathcal{L}_3(s) + \mathcal{L}_3(\varphi_{\mu\nu}) + \mathcal{L}_3(A_\mu) + \mathcal{L}_4^{(0)} + \mathcal{L}_4^{(2)}\Big), \tag{2.1}$$

where $g_a$ denotes the determinant of the AdS metric with the signature $(-1,1,...,1)$:

$$ds^2 = \frac{1}{z_0^2}(dz_0^2 + \eta_{ij}dx^i dx^j).$$

The quadratic actions for the scalars $s^I$, the graviton and the massless vector fields on the AdS space are given by [4]

$$\mathcal{L}_2(s) = \frac{2^8}{3}\sum_I\left(-\frac{1}{2}\nabla_\mu s^I\nabla^\mu s^I - \frac{1}{2}m^2 s_I^2\right), \tag{2.2}$$

$$\mathcal{L}_2(\varphi_{\mu\nu}) = -\frac{1}{4}\nabla_\rho\varphi_{\mu\nu}\nabla^\rho\varphi^{\mu\nu} + \frac{1}{2}\nabla_\mu\varphi^{\mu\rho}\nabla^\nu\varphi_{\nu\rho} - \frac{1}{2}\nabla_\mu\varphi_\rho^\rho\nabla_\nu\varphi^{\mu\nu} \\ + \frac{1}{4}\nabla_\rho\varphi_\mu^\mu\nabla^\rho\varphi_\nu^\nu + \frac{1}{2}\varphi_{\mu\nu}\varphi^{\mu\nu} + \frac{1}{2}(\varphi_\mu^\mu)^2, \tag{2.3}$$

$$\mathcal{L}_2(A_\mu) = -\frac{1}{12}\sum_I(F_{\mu\nu}(A^I))^2. \tag{2.4}$$

Here the field strength $F_{\mu\nu}(A^I)$ is defined by $F_{\mu\nu}(A^I) = \partial_\mu A_\nu^I - \partial_\nu A_\mu^I$, where $A_\mu^I$ with $I = 1, ..., 15$ represent 15 massless vectors that correspond to the Killing vectors of $S^5$. All these fields occur in the bosonic part of the massless graviton multiplet of compactified type IIB supergravity on $AdS_5 \times S^5$.

The relevant cubic terms can be easily extracted from [5, 6, 7], and they are given by

$$\mathcal{L}_3(s) = \frac{5\cdot 2^{11}}{3^3}a_{I_1I_2I_3}\ s^{I_1}s^{I_2}s^{I_3}, \tag{2.5}$$

$$\mathcal{L}_3(\varphi_{\mu\nu}) = \frac{2^7}{3\pi^{3/2}}\left(\nabla^\mu s^I\nabla^\nu s^I\varphi_{\mu\nu} - \frac{1}{2}\left(\nabla^\mu s^I\nabla_\mu s^I - 4s^I s^I\right)\varphi_\nu^\nu\right) \tag{2.6}$$

$$\mathcal{L}_3(A_\mu) = \frac{2^8}{3^2}t_{I_1I_2I_3}\ s^{I_1}\nabla^\mu s^{I_2}A_\mu^{I_3}. \tag{2.7}$$

Here the summation over $I_1,\ I_2,\ I_3$ running over the basis of irrep. **20** of $SO(6)$ is assumed, and we use the following notations

$$a_{I_1I_2I_3}=\int\ Y^{I_1}Y^{I_2}Y^{I_3},\quad t_{I_1I_2I_3}=\int\ \nabla^{\alpha}Y^{I_1}Y^{I_2}Y_{\alpha}^{I_3},$$

where the scalar $Y^I$ and the vector $Y_\alpha^I$ spherical harmonics [2] of $S^5$ satisfy $\nabla_\alpha^2 Y^I=-12Y^I$, $(\nabla_\gamma^2-4)Y_\alpha^I=-8Y_\alpha^I$. We also assumed that the spherical harmonics of different types are orthonormal, i.e. $\int Y^IY^J=\delta^{IJ}$ and $\int Y_\alpha^IY_\alpha^J=\delta^{IJ}$.

Finally, in [25] the following values of the quartic couplings of the 2-derivative vertex

$$\mathcal{L}_4^{(2)}=\frac{5^2\cdot 2^9}{27}\sum_{I_5}a_{I_1I_2I_5}a_{I_3I_4I_5}\nabla_\mu(s^{I_1}s^{I_2})\nabla^\mu(s^{I_3}s^{I_4})+\frac{2^{13}}{27\pi^3}\nabla_\mu(s^{I_1}s^{I_1})\nabla^\mu(s^{I_2}s^{I_2}) \tag{2.8}$$

and of the non-derivative vertex

$$\mathcal{L}_4^{(0)}=-\frac{5^2\cdot 2^{11}}{9}\sum_{I_5}a_{I_1I_2I_5}a_{I_3I_4I_5}s^{I_1}s^{I_2}s^{I_3}s^{I_4} \tag{2.9}$$

were found.

The quartic action can be further simplified by substituting the integrals of spherical harmonics for their explicit value via $C$-tensors (see Appendix A). Indeed, by using (5.1) together with summation formula (5.3) one gets

$$\sum_{I_5}a_{I_1I_2I_5}a_{I_3I_4I_5}=\frac{2^4\cdot 3}{5^2\pi^3}\left(C^{I_1I_2I_3I_4}+C^{I_1I_2I_4I_3}-\frac{1}{3}\delta^{I_1I_2}\delta^{I_3I_4}\right). \tag{2.10}$$

where the shorthand notation $C^{I_1I_2I_3I_4}=C^{I_1}_{i_1i_2}C^{I_2}_{i_2i_3}C^{I_3}_{i_3i_4}C^{I_4}_{i_4i_1}$ for the trace product of four matrices $C^I$ was introduced.

By using this formula, the two-derivative Lagrangian may be reduced to the following form:

$$\mathcal{L}_4^{(2)}=\frac{2^{14}}{3^2\pi^3}C_{I_1I_2I_3I_4}\nabla_\mu(s^{I_1}s^{I_2})\nabla^\mu(s^{I_3}s^{I_4}). \tag{2.11}$$

From the cubic couplings one can see that except the self-interaction, the scalars from the massless multiplet interact only via exchange by the massless graviton $\varphi_{\mu\nu}$ and by the massless vector fields $A_\mu^I$. Introduce a concise notation

$$S(s)=\frac{N^2}{8\pi^2}\int d^5x\ \sqrt{-g_a}\mathcal{L}_{red}, \tag{2.12}$$

[2]In this Section $\alpha$ is used to denote the index of $S^5$.

where the subscript in $\mathcal{L}_{red}$ stands to remind that action $S$ is obtained by dimensional reduction, and we have emphasized the 5-dimensional gravitational coupling $2\kappa_5^2 = \frac{8\pi^2}{N^2}$.

Substituting in (2.5)-(2.7) explicit values (5.1) of $a_{I_1 I_2 I_3}$ and $t_{I_1 I_2 I_3}$, using for $\mathcal{L}_4^{(0)}$ summation formula (2.10), and rescaling the fields as

$$s^I \to \frac{3^{1/2}\pi^{3/2}}{2^{9/2}} s^I, \quad A^I_\mu \to 6^{1/2}\pi^{3/2} A^I_\mu, \quad \varphi_{\mu\nu} \to \pi^{3/2}\varphi_{\mu\nu},$$

we get the Lagrangian

$$\begin{aligned} \mathcal{L}_{red} = & -\frac{1}{4}\left(\nabla_\mu s^I \nabla^\mu s^I - 4 s^I s^I\right) + \frac{1}{3} C_{I_1 I_2 I_3} s^{I_1} s^{I_2} s^{I_3} \\ & + \frac{1}{4}\left(\nabla^\mu s^I \nabla^\nu s^I \varphi_{\mu\nu} - \frac{1}{2}\left(\nabla^\mu s^I \nabla_\mu s^I - 4 s^I s^I\right)\varphi^\nu_\nu\right) \\ & + \frac{1}{2^4} C_{I_1 I_2 I_3 I_4} \nabla_\mu (s^{I_1} s^{I_2}) \nabla^\mu (s^{I_3} s^{I_4}) - \frac{3}{2^2} C_{I_1 I_2 I_3 I_4} s^{I_1} s^{I_2} s^{I_3} s^{I_4} + \frac{1}{2^3} s^{I_1} s^{I_1} s^{I_2} s^{I_2} \\ & + T_{I_1 I_2 I_3} s^{I_1} \nabla^\mu s^{I_2} A^{I_3}_\mu - \frac{1}{2} F^I_{\mu\nu} F^{\mu\nu\ I} + \mathcal{L}_2(\varphi_{\mu\nu}) \end{aligned} \tag{2.13}$$

that will be used in Section 4 to compute the 4-point functions of the lowest weight CPOs.

Finally we put this Lagrangian in the form most suitable for comparison with the relevant part of the action of the gauged $\mathcal{N} = 8$ 5d supergravity. Introducing the matrices

$$\Lambda = (\Lambda)_{ij} = C^I_{ij} s^I, \quad A_\mu = (A_\mu)_{ij} = -C^I_{i;j} A^I_\mu,$$

where $C^I_{ij}$ and $C^I_{i;j}$ are described in the Appendix A, one obtains

$$\begin{aligned} \mathcal{L}_{red} = & -\frac{1}{4}\,\mathrm{tr}\,\left(\nabla_\mu\Lambda\nabla^\mu\Lambda - 4\Lambda^2\right) + \frac{1}{3}\,\mathrm{tr}\,\Lambda^3 \\ & + \frac{1}{4}\left(\mathrm{tr}\,\nabla_\mu\Lambda\nabla_\nu\Lambda - \frac{1}{2} g_{\mu\nu}\,\mathrm{tr}\,\left(\nabla_\gamma\Lambda\nabla^\gamma\Lambda - 4\Lambda^2\right)\right)\varphi^{\mu\nu} \\ & + \frac{1}{2^4}\,\mathrm{tr}\,\left(\nabla_\mu\Lambda^2\nabla^\mu\Lambda^2\right) - \frac{3}{2^2}\,\mathrm{tr}\,\Lambda^4 + \frac{1}{2^3}\left(\mathrm{tr}\,\Lambda^2\right)^2 \\ & + \frac{1}{2}\,\mathrm{tr}\,F_{\mu\nu}F^{\mu\nu} - 2\,\mathrm{tr}\,(\nabla^\mu\Lambda\Lambda A_\mu) + \mathcal{L}_2(\varphi_{\mu\nu}), \end{aligned} \tag{2.14}$$

where $\mathrm{tr}\,F_{\mu\nu}F^{\mu\nu} = -F^{ij}_{\mu\nu}F^{\mu\nu\ ij}$ and normalization condition (5.2) was used.

# 3 Lagrangian of gauged 5d supergravity

Gauged $\mathcal{N} = 8$ five-dimensional supergravity was constructed in [26, 27] by gauging abelian vector fields of the $\mathcal{N} = 8$ Poincaré supergravity. The gauged theory has a local non-abelian

$SO(6)$ symmetry, a local composite $USp(8)$ symmetry and a global $SL(2,\mathbf{R})$ symmetry. The bosonic field content is given by graviton, fifteen real vector fields $A_{\mu\ ij}$, $i,j=1,\dots 6$ transforming in the adjoint representation of $SO(6)$, 12 antisymmetric tensors of the second rank and by 42 scalars that in the ungauged theory parametrize the non-compact manifold $E_{6(6)}/USp(8)$. In what follows we adopt the conventions of [28].

Let $A,B,...=1,\dots 8$ be the indices of the representation $\mathbf{27}$ of $E_{6(6)}$ and $a,b,...$ be $USp(8)$ indices that are raised and lowered with the symplectic metric $\Omega_{ab}$. Explicitly, an element of $E_{6(6)}/USp(8)$ can be described by the scalar vielbein $V_{AB}{}^{ab}$ which is $27\times 27$. In the gauged theory minimal couplings of the connection $A_{\mu\ ij}$ responsible for the local $SO(6)$ symmetry are introduced to all the fields transforming linearly under $SO(6)$. The transformation properties of the fields under $SO(6)$ are then uniquely specified by the embedding of $SO(6)$ into the group $SL(6,\mathbf{R})$, the latter being a subgroup of $E_{6(6)}$. Recall that under the subgroup $SL(6,\mathbf{R})\times SL(2,\mathbf{R})$ the representation $\mathbf{27}$ of $E_{6(6)}$ is decomposed as $27=(15,1)+(6,2)$. The components of the vielbein are then denoted as $V_{ij}{}^{ab}$ and $V_{i\alpha}{}^{ab}$, where $i,j=1,\dots 6$ are $SL(6,\mathbf{R})$ and $\alpha=1,2$ are $SL(2,\mathbf{R})$ indices.

The relevant bosonic part[3] of the Lagrangian of the gauged 5d gravity is of the form

$$\mathcal{L}=R-\frac{1}{6}P_{\mu\ abcd}P_{\mu}{}^{abcd}-P-\frac{1}{2}F_{\mu\nu;ij}F^{\mu\nu;ij}. \tag{3.1}$$

Here $F_{\mu\nu;ij}$ is a $SO(6)$-covariant Yang-Mills field strength, $P$ is a scalar potential and the tensor $P_{\mu abcd}$ is given by

$$P_{\mu\ ab}{}^{cd}=(V^{-1})^{cd\ AB}\nabla_{\mu}V_{AB\ ab}+2Q_{\mu[a}^{[c}\delta_{b]}^{d]}-2g(V^{-1})^{cd\ ij}A_{\mu i}{}^{k}V_{kj\ ab}-g(V^{-1})^{cd\ i\alpha}A_{\mu i}{}^{j}V_{j\alpha\ ab}$$

and it represents a coset element in the decomposition of the $E_{6(6)}$ Lie algebra into an $USp(8)$ and a coset part. In particular, matrix $Q_{\mu[a}{}^{[c}\delta_{b]}^{d]}=\sum_{k=1}^{36}B_{\mu}^{k}(T^{k})_{ab}^{cd}$ is an $USp(8)$-connection responsible for the local $USp(8)$ symmetry. Recall that $USp(8)$-connection $B_{\mu}^{k}$ is non-dynamical since it does not have a kinetic term. Therefore, it can be excluded by using its equation of motion as in fact is done below. The dimension of $USp(8)$ is 36 and $T^k$ is a basis of the $\mathbf{27}$ irrep. of the $USp(8)$ Lie algebra, $g$ is the Yang-Mills coupling constant.

Eq.(3.1) is our starting point to find the action for scalars $s^I$ on the $AdS_5$ background. Since the potential for $s^I$ was already found in studying the critical points the only missing piece is an explicit construction of the kinetic term.

To build the kinetic term we need an explicit parametrization of the scalar vielbein in terms of 20 scalar fields that are neutral under $SL(2,\mathbf{R})$. We then employ the parametrization of

[3]We put all antisymmetric fields to zero, and changed the overall normalization of the Lagrangian in comparison to [28].

[28], in which 42 scalars are represented by two real symmetric traceless matrices $\Lambda_i{}^j$ and $\Lambda_\alpha{}^\beta$, $\alpha,\beta = 1,2$ and by a real completely antisymmetric in $i,\ j,\ k$ tensor $\phi_{ijk\alpha}$ obeying the self-duality condition

$$\phi_{ijk\alpha} = \frac{1}{6}\varepsilon_{\alpha\beta}\varepsilon_{ijklmn}\phi_{lmn\beta}.$$

Since only $\Lambda$ is a singlet under $SL(2,\mathbf{R})$ in what follows we put $\Lambda_\alpha{}^\beta$ and $\phi_{ijk\alpha}$ to zero. Turning off these fields is allowed in our specific problem of constructing the action for $s^I$ because the existence of the cubic terms containing two $SL(2,\mathbf{R})$-singlets $s^I$ and one doublet field is forbidden by the $SL(2,\mathbf{R})$ symmetry. In fact, from the point of view of the dimensional reduction of type IIB supergravity matrix $\Lambda_\alpha{}^\beta$ describes zero modes of axion and dilaton fields while $\phi_{ijk\alpha}$ encodes the scalars arising from the reduction of the antisymmetric tensor fields.

With this parametrization at hand we get the following expression for the vielbein $V_{AB}^{ab}$ in the $SL(6,\mathbf{R})\times SL(2,\mathbf{R})$ basis:

$$\begin{aligned} V^{ij\ ab} &= \frac{1}{4}(\Gamma_{kl})^{ab}S^i_kS^j_l, \quad (V^{-1})_{ij\ cd} = \frac{1}{4}(\Gamma_{kl})_{cd}(S^{-1})^k_i(S^{-1})^l_j, \\ V_{i\alpha}{}^{ab} &= \frac{1}{2^{3/2}}(\Gamma_{k\alpha})^{ab}S^k_i, \quad (V^{-1})^{i\alpha}_{cd} = -\frac{1}{2^{3/2}}(\Gamma_{k\alpha})_{cd}(S^{-1})^i_k, \end{aligned} \tag{3.2}$$

where $\Gamma$ are $SO(6)$ $\Gamma$-matrices (see Appendix A) and $S$ is given by $S = e^\Lambda$ with $\Lambda$ being the traceless symmetric $6\times 6$-matrix comprising 20 scalars.

It is convenient to introduce a matrix $R_\mu$:

$$R_\mu = \nabla_\mu SS^{-1} + gSA_\mu S^{-1}. \tag{3.3}$$

Since $\Lambda$ is traceless and $A_{\mu i}^{\ j}$ is antisymmetric this matrix appears to be traceless: $R_{\mu i}{}^i = 0$.

The scalar kinetic part of Lagrangian (3.1) in parametrization (3.2) is then computed in the Appendix A and the result looks as follows

$$P_{\mu abcd}P_\mu^{abcd} = \frac{3}{2}\,\mathrm{tr}\,\left(R_\mu + R_\mu^t\right)^2 .$$

Substituting the potential found in [28], we get the final answer for the Lagrangian (for simplicity we omit for the moment the gravity and the gauge terms):

$$\mathcal{L} = -\frac{1}{4}\,\mathrm{tr}\,\left(R_\mu + R_\mu^t\right)^2 + \frac{g^2}{8}\left((\,\mathrm{tr}\,SS\,)^2 - 2\,\mathrm{tr}\,(SSSS)\right). \tag{3.4}$$

Scalar fields $\Lambda_i{}^j$ transform in the **20** of $SO(6)$. We are interested in the maximally supersymmetric vacuum with only non-trivial bosonic fields, which implies that the background solution is invariant under $SO(6)$. Thus, at the $SO(6)$ invariant critical point $P_0$ of the potential the

scalar fields should acquire some expectation values that are invariant under $SO(6)$. Clearly, the only possibility for that is to take $\Lambda_i^{\ j} = 0$, i.e., to put $S$ to be the unit matrix. The value of the potential is then $P_0 = -\frac{3}{4}g^2$ that leads to the equation of motion:

$$R_{\mu\nu} = \frac{4}{3}P_0 = -g^2 g_{\mu\nu}.$$

Thus, the background solution is the anti-de Sitter space with the cosmological constant $\lambda = -\frac{3}{2}g^2$ and with vanishing scalars $\Lambda_i^{\ j}$. Decomposition of Lagrangian (3.4) near this background is then easily obtained by decomposing $S = e^{\Lambda}$ around $\Lambda = 0$.

We find up to the cubic order

$$\begin{aligned}
\nabla_\mu S S^{-1} &= \nabla_\mu \Lambda - \frac{1}{2}(\nabla_\mu \Lambda\Lambda - \Lambda\nabla_\mu \Lambda) - \frac{1}{2}\Lambda\nabla_\mu\Lambda\Lambda + \frac{1}{6}\nabla_\mu \Lambda^3, \\
(\nabla_\mu S S^{-1})^t &= \nabla_\mu \Lambda + \frac{1}{2}(\nabla_\mu \Lambda\Lambda - \Lambda\nabla_\mu \Lambda) - \frac{1}{2}\Lambda\nabla_\mu\Lambda\Lambda + \frac{1}{6}\nabla_\mu \Lambda^3.
\end{aligned}$$

By using these formulae, one then gets

$$R_\mu + R_\mu^t = 2\nabla_\mu \Lambda - \Lambda\nabla_\mu\Lambda\Lambda + \frac{1}{3}\nabla_\mu\Lambda^3 + 2g[\Lambda, A_\mu]. \tag{3.5}$$

The terms quadratic in $\Lambda$ cancelled and, therefore, the action does not contain cubic in $\Lambda$ terms with two derivatives.

Analogously, for the potential we find

$$\frac{g^2}{8}\Big((\operatorname{tr} SS)^2 - 2\operatorname{tr}(SSSS)\Big) = g^2\left(3 + \operatorname{tr}\Lambda^2 - \frac{2}{3}\operatorname{tr}\Lambda^3 - \frac{5}{3}\operatorname{tr}\Lambda^4 + \frac{1}{2}(\operatorname{tr}\Lambda^2)^2\right).$$

To compare action (3.1) with the one from the previous Section we have to fix the coupling constant $g$. It is fixed to be $g^2 = 4$ by the requirement to have the vacuum solution defined by the equation $R_{\mu\nu} = -4g_{\mu\nu}$. Namely this background solution was used to obtain the action (2.14) by compactifying ten-dimensional type IIB supergravity.

Thus, for Eq.(3.4) up to the fourth order in $\Lambda$ we get

$$\begin{aligned}
\mathcal{L} = 12 - &\operatorname{tr}\left(\nabla_\mu\Lambda\nabla^\mu\Lambda - 4\Lambda^2\right) - \frac{2}{3}\operatorname{tr}(\nabla_\mu\Lambda\Lambda\Lambda\nabla^\mu\Lambda - \Lambda\nabla_\mu\Lambda\Lambda\nabla^\mu\Lambda) \\
&- \frac{8}{3}\operatorname{tr}\Lambda^3 - \frac{20}{3}\operatorname{tr}\Lambda^4 + 2(\operatorname{tr}\Lambda^2)^2 - 8\operatorname{tr}(\nabla^\mu\Lambda\Lambda A_\mu)
\end{aligned}$$

It is then useful to perform the following field redefinition

$$\Lambda \to \Lambda + r\Lambda^3$$

under which the Lagrangian transforms into

$$\begin{aligned}\mathcal{L} =\ & R + 12 - \operatorname{tr}\left(\nabla_\mu\Lambda\nabla^\mu\Lambda - 4\Lambda^2\right) \\ & - 4\operatorname{tr}\left(\left(\frac{1}{6}+r\right)\nabla_\mu\Lambda\Lambda\Lambda\nabla^\mu\Lambda + \left(\frac{r}{2}-\frac{1}{6}\right)\Lambda\nabla_\mu\Lambda\Lambda\nabla^\mu\Lambda\right) \\ & - \frac{8}{3}\operatorname{tr}\Lambda^3 + 4\left(-\frac{5}{3}+2r\right)\operatorname{tr}\Lambda^4 + 2(\operatorname{tr}\Lambda^2)^2 \\ & + \frac{1}{2}\operatorname{tr}F_{\mu\nu}F^{\mu\nu} - 8\operatorname{tr}\left(\nabla^\mu\Lambda\Lambda A_\mu\right),\end{aligned}$$

where we have restored the gravity and gauge terms. Let us choose $r$ to be $r = -2/3$. Then taking into account that

$$\operatorname{tr}\left(\nabla_\mu\Lambda^2\nabla^\mu\Lambda^2\right) = 2\operatorname{tr}\left(\nabla_\mu\Lambda\Lambda\Lambda\nabla_\mu\Lambda + \Lambda\nabla_\mu\Lambda\Lambda\nabla_\mu\Lambda\right),$$

and making the rescaling $\Lambda \to -\frac{1}{2}\Lambda$, we find

$$\begin{aligned}\mathcal{L} =\ & R + 12 - \frac{1}{4}\operatorname{tr}\left(\nabla_\mu\Lambda\nabla^\mu\Lambda - 4\Lambda^2\right) + \frac{1}{3}\operatorname{tr}\Lambda^3 \\ & + \frac{1}{2^4}\operatorname{tr}\left(\nabla_\mu\Lambda^2\nabla^\mu\Lambda^2\right) - \frac{3}{2^2}\operatorname{tr}\Lambda^4 + \frac{1}{2^3}\left(\operatorname{tr}\Lambda^2\right)^2 \\ & + \frac{1}{2}\operatorname{tr}F_{\mu\nu}F^{\mu\nu} - 2\operatorname{tr}\left(\nabla^\mu\Lambda\Lambda A_\mu\right).\end{aligned} \tag{3.6}$$

Note that $-6$ is the cosmological constant in the action $\int d^{d+1}x\sqrt{-g}(R-2\lambda)$, $\lambda = -\frac{1}{2}d(d-1)$ for $d=4$ that appears in the reduction from ten dimensions.

Multiplying (3.6) by $\sqrt{-g}$, and decomposing the metric $g_{\mu\nu} = g^0_{\mu\nu} + \varphi_{\mu\nu}$ near the background AdS solution $g^0_{\mu\nu}$, one immediately finds

$$\mathcal{L} = \mathcal{L}_{red}.$$

Thus, we have shown that the action for the scalars $s^I$ obtained by compactification of type IIB supergravity on $AdS_5 \times S^5$ with further reduction to the fields from the massless graviton multiplet coincides with the relevant part of the action of the gauged $\mathcal{N}=8$ five-dimensional supergravity on $AdS_5$ background.

# 4 4-point function of lowest weight CPOs

The normalized lowest weight CPOs in $\mathcal{N}=4$ SYM$_4$ are operators of the form

$$O^I(\vec{x}) = \frac{2^{3/2}\pi^2}{\lambda}C^I_{ij}\operatorname{tr}(:\phi^i\phi^j:).$$

By using the following propagator $\langle \phi^i_a \phi^j_b \rangle = \frac{g^2_{YM}\delta_{ab}\delta^{ij}}{(2\pi)^2 x^2_{12}}$, where $a, b$ are color indices and $x_{ij} = \vec{x}_i - \vec{x}_j$, one finds in the free approximation and at leading order in $1/N$ the following expressions for 2-, 3- [5] and 4-point functions of $O^I$:

$$\begin{aligned}
&\langle O^{I_1}(\vec{x}_1) O^{I_2}(\vec{x}_2) \rangle = \frac{\delta^{I_1 I_2}}{x^2_{12}}, \\
&\langle O^{I_1}(\vec{x}_1) O^{I_2}(\vec{x}_2) O^{I_3}(\vec{x}_3) \rangle = \frac{1}{N} \frac{2^{3/2} C^{I_1 I_2 I_3}}{x^2_{12} x^2_{13} x^2_{23}}, \\
&\langle O^{I_1}(\vec{x}_1) O^{I_2}(\vec{x}_2) O^{I_3}(\vec{x}_3) O^{I_4}(\vec{x}_4) \rangle = \frac{\delta^{I_1 I_2}\delta^{I_3 I_4}}{x^4_{12} x^4_{34}} + \frac{1}{N^2} \frac{4 C_{I_1 I_2 I_3 I_4}}{x^2_{12} x^2_{14} x^2_{23} x^2_{34}} + \text{permutations},
\end{aligned} \tag{4.1}$$

where the first term in the 4-point function represents the contribution of disconnected diagrams.

In this Section we compute 4-point functions of $O^I$ from AdS supergravity. The starting point is action (2.13). We will work with the Euclidean version of $AdS_5$ that amounts to changing in (2.13) an overall sign, so that

$$\begin{aligned}
\mathcal{L}_{red} = &\frac{1}{4}\left(\nabla_\mu s^I \nabla^\mu s^I - 4 s^I s^I\right) - \frac{1}{3} C_{I_1 I_2 I_3} s^{I_1} s^{I_2} s^{I_3} \\
&- \frac{1}{4}\left(\nabla^\mu s^I \nabla^\nu s^I \varphi_{\mu\nu} - \frac{1}{2}\left(\nabla^\mu s^I \nabla_\mu s^I - 4 s^I s^I\right)\varphi^\nu_\nu\right) \\
&- \frac{1}{2^4} C_{I_1 I_2 I_3 I_4} \nabla_\mu (s^{I_1} s^{I_2}) \nabla^\mu (s^{I_3} s^{I_4}) + \frac{3}{2^2} C_{I_1 I_2 I_3 I_4} s^{I_1} s^{I_2} s^{I_3} s^{I_4} - \frac{1}{2^3} s^{I_1} s^{I_1} s^{I_2} s^{I_2} \\
&- T_{I_1 I_2 I_3} s^{I_1} \nabla^\mu s^{I_2} A^{I_3}_\mu + \frac{1}{2} F^I_{\mu\nu} F^{\mu\nu\ I} - \mathcal{L}_2(\varphi_{\mu\nu})
\end{aligned} \tag{4.2}$$

It is convenient to introduce the following currents

$$\begin{aligned}
T_{\mu\nu} &= \nabla_\mu s^I \nabla_\nu s^I - \frac{1}{2} g_{\mu\nu}\left(\nabla^\rho s^I \nabla_\rho s^I - 4 s^I s^I\right), \\
J^{I_3}_\mu &= T_{I_1 I_2 I_3}(s^{I_1} \nabla_\mu s^{I_2} - s^{I_2} \nabla_\mu s^{I_1}),
\end{aligned}$$

both of them are conserved on-shell: $\nabla^\mu T_{\mu\nu} = \nabla^\mu J^I_\mu = 0$.

From (4.2) we get the following equations of motion:

**1**. for scalars $s^I$:

$$(\nabla^2_\mu - m^2) s^I = -2 C_{IJK} s^J s^K; \tag{4.3}$$

**2**. for vector fields $A^I_\mu$:

$$\nabla^\nu (\nabla_\nu A^I_\mu - \nabla_\mu A^I_\nu) = -\frac{1}{4} J^I_\mu; \tag{4.4}$$

**3**. for the graviton $\varphi_{\mu\nu}$:

$$W_{\mu\nu}^{\ \ \rho\lambda}\varphi_{\rho\lambda} = \frac{1}{4}\left(g_{\mu\mu'}g_{\nu\nu'} + g_{\mu\nu'}g_{\nu\mu'} - \frac{2}{3}g_{\mu\nu}g_{\mu'\nu'}\right)T^{\mu'\nu'}, \tag{4.5}$$

where $W_{\mu\nu}^{\ \ \rho\lambda}$ is the Ricci operator

$$W_{\mu\nu}^{\ \ \rho\lambda}\varphi_{\rho\lambda} = -\nabla_\rho^2\varphi_{\mu\nu} + \nabla_\mu\nabla^\rho\varphi_{\rho\nu} + \nabla_\nu\nabla^\rho\varphi_{\rho\mu} - \nabla_\mu\nabla_\nu\varphi_\rho^\rho - 2(\varphi_{\mu\nu} - g_{\mu\nu}\varphi_\rho^\rho).$$

Introduce the scalar $G$ [30], the vector $G_{\mu\nu}$ and the graviton $G_{\mu\nu\ \rho\lambda}$ [15] propagators

$$\begin{aligned}
&(\nabla_a^2 - m^2)G(u) = -\delta(z,w),\\
&\nabla^\rho(\nabla_\rho G^I_{\mu\nu} - \nabla_\mu G^I_{\nu\rho}) = -g_{\mu\nu}\delta(z,w),\\
&W_{\mu\nu}^{\ \ \rho\lambda}G_{\rho\lambda\ \mu'\nu'} = \left(g_{\mu\mu'}g_{\nu\nu'} + g_{\mu\nu'}g_{\nu\mu'} - \frac{2}{3}g_{\mu\nu}g_{\mu'\nu'}\right)\delta(z,w)
\end{aligned}$$

being the functions of the invariant AdS-distance $u$:

$$u = \frac{(z-w)^2}{2z_0w_0}, \qquad (z-w)^2 = \delta_{\mu\nu}(z-w)_\mu(z-w)_\nu.$$

Represent the solution to the equations of motion in the form

$$s = s^0 + s^1, \quad A_\mu = A_\mu^0 + A_\mu^1, \quad \varphi_{\mu\nu} = \varphi_{\mu\nu}^0 + \varphi_{\mu\nu}^1,$$

where $s^0$, $A_\mu^0$ and $\varphi_{\mu\nu}^0$ are solutions of the linearized equations with fixed boundary conditions and $s^1$, $A_\mu^1$ and $\varphi_{\mu\nu}^1$ are the corrections with vanishing boundary conditions. Then by perturbation theory for $s^1$, $A_\mu^1$ and $\varphi_{\mu\nu}^1$ one gets

$$\begin{aligned}
s_I^1(w) &= 2C_{IJK}\int\frac{d^5z}{z_0^5}G(u)s^J(z)s^K(z),\\
A_\mu^{1\ I}(w) &= \frac{1}{4}\int\frac{d^5z}{z_0^5}G_\mu^{\ \nu}(u)J_\nu^I(z),\\
\varphi_{\mu\nu}^1(w) &= \frac{1}{4}\int\frac{d^5z}{z_0^5}G_{\mu\nu\ \mu'\nu'}(u)T^{\mu'\nu'}(z),
\end{aligned} \tag{4.6}$$

where the r.h.s. depends only on $s^0$, $A_\mu^0$ and $\varphi_\mu^0$ and from now on we omit the superscript 0 unless we want to indicate explicitly that we deal with solutions of the linearized equations of motion.

It is worth noting that not only the interaction terms but also the quadratic action $\mathcal{L}_{quad}$ gives a contribution to the on-shell value of action (2.13) depending quartically on $s_0$:

$$\mathcal{L}_{quad} = \frac{1}{2}C_{IJK}s_0^Is_0^Js_1^K + \frac{1}{8}\varphi_{\mu\nu}^1T^{\mu\nu} + \frac{1}{4}A_\mu^{1\ I}J^{\mu\ I}.$$

Taking into account the summation formula

$$\sum_{I_5} T_{I_1I_2I_5}T_{I_3I_4I_5} = 2\left(C_{I_1I_2I_4I_3} - C_{I_1I_2I_3I_4}\right), \tag{4.7}$$

that follows from (5.4) and using (4.3) we arrive at the following expression for the on-shell value of (4.2):

$$\begin{aligned}\mathcal{L}_{red} = & \frac{1}{4}C_{I_1I_2I_3I_4}\int \frac{d^5z}{z_0^5} s^{I_1} \overleftrightarrow{\nabla}^{\mu} s^{I_2}(w)G_{\mu\nu}(u)s^{I_3} \overleftrightarrow{\nabla}^{\nu} s^{I_4}(z) \\ & - \frac{1}{2^5}\int \frac{d^5z}{z_0^5} T^{\mu\nu}(w)G_{\mu\nu\ \rho\lambda}(u)T^{\rho\lambda}(z) \\ & - \left(C_{I_1I_2I_3I_4} - \frac{1}{6}\delta_{I_1I_2}\delta_{I_3I_4}\right)\int \frac{d^5z}{z_0^5} G(u)s^{I_1}(w)s^{I_2}(w)s^{I_3}(z)s^{I_4}(z) \\ & - \frac{1}{2^4}C_{I_1I_2I_3I_4}\nabla_{\mu}(s^{I_1}s^{I_2})\nabla^{\mu}(s^{I_3}s^{I_4}) + \frac{3}{4}C_{I_1I_2I_3I_4}s^{I_1}s^{I_2}s^{I_3}s^{I_4} - \frac{1}{8}s^{I_1}s^{I_1}s^{I_2}s^{I_2}.\end{aligned}$$

On the language of the Feynman diagrams the first three terms here involving $z$-integrals describe the exchange by the gauge boson, by the graviton and by the scalar fields respectively. The other contributions correspond to contact diagrams. $z$-integrals are easily computed by the technique of [17] and in the Appendix B we list the corresponding results. It is worthwhile to note that since we compute the on-shell value of the gravity action, we take into account only the connected $AdS$ graphs.

Recall that the solution of the Dirichlet boundary problem for the scalar field $s^I$ of mass $m^2 = -4$ on $AdS_5$ reads as

$$s^I(z,\vec{x}) = \frac{1}{2\pi^2}\int d^4\vec{x}K_2(w,\vec{x})s^I(\vec{x}), \tag{4.8}$$

where $s^I(\vec{x})$ is a boundary value and

$$K_{\Delta}(w,\vec{x}) = \left(\frac{w_0}{w_0^2 + (\vec{w}-\vec{x})^2}\right)^{\Delta}.$$

With this normalization of the bulk-to-boundary propagator the two-point function of corresponding boundary operators appears to be finite in the limit when the AdS cut-off $\varepsilon$ tends to zero (see Appendix B for details).

Introducing the notation

$$D_{\Delta_1\Delta_2\Delta_3\Delta_4} = \int \frac{d^5w}{w_0^5} K_{\Delta_1}(w,\vec{x}_1)K_{\Delta_2}(w,\vec{x}_2)K_{\Delta_3}(w,\vec{x}_3)K_{\Delta_4}(w,\vec{x}_4) \tag{4.9}$$

and using identities for $D$-functions (see Appendix B) we find the following on-shell value for (2.13):

$$
\begin{aligned}
S = \frac{N^2}{8\pi^2} \int d^4x_1 d^4x_2 d^4x_3 d^4x_4 s^{I_1}(\vec{x}_1) s^{I_2}(\vec{x}_2) s^{I_3}(\vec{x}_3) s^{I_4}(\vec{x}_4) \Big( & \qquad (4.10) \\
\frac{1}{2^7\pi^8} C^-_{I_1I_2I_3I_4} \frac{1}{x_{12}^2 x_{34}^2} \left(2(x_{13}^2x_{24}^2 - x_{14}^2x_{23}^2)D_{2222} - x_{24}^2 D_{1212} - x_{13}^2 D_{2121} + x_{14}^2 D_{2112} + x_{23}^2 D_{1221}\right) & \\
-\frac{1}{2^7\pi^8} \delta^{I_1I_2}\delta^{I_3I_4} \left( -\frac{1}{2x_{34}^2} D_{2211} + \frac{(x_{13}^2x_{24}^2 + x_{14}^2x_{23}^2 - x_{12}^2x_{34}^2)}{x_{34}^2} D_{3322} + \frac{3}{2} D_{2222} \right) & \\
-\frac{1}{2^6\pi^8}\, C^+_{I_1I_2I_3I_4} \left( \frac{1}{x_{34}^2} D_{2211} + 4x_{34}^2 D_{2233} - 3D_{2222} \right) \Big), &
\end{aligned}
$$

where $C^{\pm}_{I_1I_2I_3I_4} = \frac{1}{2}(C_{I_1I_2I_3I_4} \pm C_{I_2I_1I_3I_4})$. The expression under the integral represents the contribution of the s-channel since it possesses the s-channel symmetries $1 \leftrightarrow 2$, $3 \leftrightarrow 4$ and $(12) \leftrightarrow (34)$. In the expression for the 4-point function the t-channel contribution is obtained from this one by the interchange $1 \leftrightarrow 4$ and the u-channel one by $1 \leftrightarrow 3$.

Taking into account the normalization of the quadratic part of (4.2) and formula (6.3) from the Appendix B, we get the 2-point function of unnormalized CPOs $\mathcal{O}^I$:

$$
\langle \mathcal{O}^I(\vec{x}_1)\mathcal{O}^J(\vec{x}_2)\rangle = \frac{N^2}{2^5\pi^4}\frac{\delta^{IJ}}{x_{12}^4}. \qquad (4.11)
$$

Introducing then the normalized CPOs as $O^I = \frac{(2^5\pi^4)^{1/2}}{N}\mathcal{O}^I$, we obtain from (4.10) the following 4-point function of the normalized CPOs:

$$
\begin{aligned}
\langle O^{I_1}(\vec{x}_1)O^{I_2}(\vec{x}_2)O^{I_3}(\vec{x}_3)O^{I_4}(\vec{x}_3)\rangle = \frac{8}{N^2\pi^2} \times & \qquad (4.12) \\
\Big( -C^-_{I_1I_2I_3I_4} \frac{1}{x_{12}^2 x_{34}^2} \left(2(x_{13}^2x_{24}^2 - x_{14}^2x_{23}^2)D_{2222} - x_{24}^2 D_{1212} - x_{13}^2 D_{2121} + x_{14}^2 D_{2112} + x_{23}^2 D_{1221}\right) & \\
+\delta^{I_1I_2}\delta^{I_3I_4} \left( -\frac{1}{2x_{34}^2} D_{2211} + \frac{(x_{13}^2x_{24}^2 + x_{14}^2x_{23}^2 - x_{12}^2x_{34}^2)}{x_{34}^2} D_{3322} + \frac{3}{2} D_{2222} \right) & \\
+2\, C^+_{I_1I_2I_3I_4} \left( \frac{1}{x_{34}^2} D_{2211} + 4x_{34}^2 D_{2233} - 3D_{2222} \right) + t + u \Big), &
\end{aligned}
$$

where $t$ and $u$ stand for the above discussed contributions of the $t$- and $u$-channels. Due to the conformal behaviour of the $D$-functions Eq.(4.12) represents a correct conformally covariant expression for a 4-point function of operators with conformal dimension $\Delta = 2$.

This set of 4-point functions allows one to approach the problem of finding the OPE of the simplest CPOs in $\mathcal{N} = 4$ SYM$_4$ that will be the subject of our further study.

# 5 Appendix A

### Integrals of spherical harmonics

Considering the action for the fields $s^I$, we need the following explicit expressions for the integrals $a_{I_1I_2I_3}$ and $t_{I_1I_2I_3}$ involving the scalar spherical harmonics $Y^I$ [4] and Killing vectors $Y^I_a$ [5, 6]:

$$a_{I_1I_2I_3} = \frac{2^2 \cdot 6^{1/2}}{5\pi^{3/2}} C_{I_1I_2I_3} \qquad t_{I_1I_2I_3} = \frac{6^{1/2}}{\pi^{3/2}} T_{I_1I_2I_3}. \tag{5.1}$$

If we introduce a basis $C^I_{ij}$ in the space of symmetric traceless second rank tensors of $SO(6)$ and a basis $C^I_{i;j}$ in the space of antisymmetric tensors with normalization conditions

$$C^I_{ij}C^J_{ij} = \delta^{IJ}, \qquad C^I_{i;k}C^J_{j;k} = \frac{1}{6}\delta^{IJ}\delta_{ij} \tag{5.2}$$

then the tensors $C_{I_1I_2I_3}$ and $T_{I_1I_2I_3}$ are given by

$$C^{I_1I_2I_3} = C^{I_1}_{ij}C^{I_2}_{jk}C^{I_3}_{ki}, \quad T^{I_1I_2I_3} = C^{I_1}_{ik}C^{I_2}_{kj}C^{I_3}_{i;j} - C^{I_1}_{jk}C^{I_2}_{ki}C^{I_3}_{i;j},$$

where we have written tensor $T^{I_1I_2I_3}$ to be explicitly antisymmetric in indices $I_1, I_2$.

One can easily establish the following summation formula

$$\sum_I C^I_{ij}C^I_{kl} = \frac{1}{2}\delta_{ik}\delta_{jl} + \frac{1}{2}\delta_{il}\delta_{jk} - \frac{1}{6}\delta_{ij}\delta_{kl} \tag{5.3}$$

that steams from the fact that the l.h.s. of the expression above is a fourth rank tensor of $SO(6)$, symmetric and traceless both in $(ij)$ and $(kl)$ indices with the normalization condition $C^I_{ij}C^I_{ij} = 20$.

Analogously one finds

$$\sum_I C^I_{m;l}C^I_{n;s} = \frac{1}{2}(\delta_{mn}\delta_{ls} - \delta_{ms}\delta_{nl}) \tag{5.4}$$

since this time the l.h.s. of (5.4) is a traceless and antisymmetric in $m, l$ and in $n, s$ indices fourth rank tensor of $SO(6)$ that agrees with the normalization (5.2).

### Some properties of $SO(6)$ $\Gamma$-matrices

In studying the action of the gauged supergravity, we need an identity that follows from the completeness condition for $SO(6)$ $\Gamma$-matrices and may be found in [26, 27, 28]. To make the treatment self-contained we recall its derivation here.

[4] They describe a basis of irrep. **20** of $SO(6)$.

Consider the Clifford algebra in $d = 6$ Euclidean dimensions:

$$\{\Gamma_i, \Gamma_j\} = 2\delta_{ij}, \quad i, j, k, l, n = 1, ..., 6.$$

The $\Gamma$-matrices can be represented by hermitian skew-symmetric $8 \times 8$ matrices $(\Gamma_i)_a^b$. Indices $a, b = 1, ..., 8$ are raised or lowered by the symmetric charge conjugation matrix $C_{ab}$ that in the chosen representation coincides with $\delta_{ab}$. Thus, we do not distinguish the upper and lower indices.

Clearly, the matrices

$$\Gamma_i, \quad i\Gamma_i\Gamma_0, \quad \Gamma_{ij}, \quad \Gamma_0 = i\Gamma_1\Gamma_2\Gamma_3\Gamma_4\Gamma_5\Gamma_6 \tag{5.5}$$

are skew-symmetric. Their number is $6 + 6 + 15 + 1 = 28$ and it coincides with a total number $8 \cdot 7/2 = 28$ of independent skew-symmetric matrices among all $8 \times 8$ matrices. Therefore, any skew-symmetric matrix $A_{ab}$ can be decomposed over the basis (5.5):

$$A_{ab} = \alpha_1^i(\Gamma_i)_{ab} + \alpha_2^i(i\Gamma_i\Gamma_0)_{ab} + \frac{1}{2}\alpha_3^{ij}(\Gamma_{ij})_{ab} + \alpha_4(\Gamma_0)_{ab}. \tag{5.6}$$

Here in the third term we assume the summation over the whole set of indices - not just over $i < j$. We also use the convention that $\alpha_3^{ij} = -\alpha_3^{ji}$. The coefficients are easy to compute

$$\alpha_1^i = \frac{1}{8}tr(A\Gamma_i), \quad \alpha_2^i = \frac{i}{8}tr(A\Gamma_i\Gamma_0), \quad \alpha_3^{ij} = -\frac{1}{8}tr(A\Gamma_{ij}), \quad \alpha_4 = \frac{1}{8}tr(A\Gamma_0).$$

Substituting these coefficients back in (5.6), and using the fact that Eq.(5.6) should hold for any skew-symmetric matrix $A_{ab}$, we find an identity:

$$\frac{1}{16}(\Gamma_{ij})_{ab}(\Gamma_{ij})_{cd} - \frac{1}{8}(\Gamma_i)_{ab}(\Gamma_i)_{cd} - \frac{1}{8}(i\Gamma_i\Gamma_0)_{ab}(i\Gamma_i\Gamma_0)_{cd} = \frac{1}{2}(\delta_{ac}\delta_{bd} - \delta_{ad}\delta_{bc}) - \frac{1}{8}(i\Gamma_0)_{ab}(i\Gamma_0)_{cd},$$

the term with $\alpha_4$ was written in the l.h.s..

If one introduces the symplectic metric $\Omega^{ab} = -i(\Gamma_0)^{ab} = -\Omega_{ab}$ and matrices $\Gamma_{i\alpha} = (\Gamma_i, i\Gamma_i\Gamma_0)$ for $\alpha = 1, 2$ then the last indentity reads as follows [26, 27, 28]:

$$\frac{1}{16}(\Gamma_{ij})_{ab}(\Gamma_{ij})^{cd} - \frac{1}{8}(\Gamma_{i\alpha})_{ab}(\Gamma_{i\alpha})^{cd} = \frac{1}{2}(\delta_a^c\delta_b^d - \delta_a^d\delta_b^c) + \frac{1}{8}\Omega_{ab}\Omega^{cd}. \tag{5.7}$$

Here in the l.h.s. we have written some indices up since the r.h.s. represents now a tensor of $USp(8)$. It is as well to note that except the symmetric charge conjugation matrix that is just the unit matrix one can also raise and lower indices with the $USp(8)$ metric $\Omega_{ab}$.

We also summarize the trace formulae needed in the paper

$$\mathrm{tr}\,(\,\Gamma_{ij}\Gamma_{kl}) = 8(\delta_{il}\delta_{kj} - \delta_{ik}\delta_{jl}), \tag{5.8}$$

$$\mathrm{tr}\,(\,\Gamma_{in}\Gamma_{jn}\Gamma_{kl}) = 32(\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}), \tag{5.9}$$

$$\mathrm{tr}\,(\,\Gamma_{i\alpha}\Gamma_{j\alpha}\Gamma_{kl}) = 16(\delta_{il}\delta_{jk} - \delta_{ik}\delta_{jl}). \tag{5.10}$$

Note that matrices $\Gamma_i$ are hermitian while $\Gamma_0$, $\Gamma_{ij}$ and $i\Gamma_i\Gamma_0$ are antihermitian. It follows from here that $\Gamma_{ij}$ and $i\Gamma_i\Gamma_0$ are real.

**Scalar kinetic part of the lagrangian of the gauged 5d supergravity**

By using (5.7), one can check the following relation:

$$(V^{-1})_{cd}^{AB} V_{AB}^{ab} = (V^{-1})_{cd\ ij} V^{ij\ ab} + (V^{-1})_{cd}^{i\alpha} V_{i\alpha}{}^{ab} = \frac{1}{2}(\delta_a^c\delta_b^d - \delta_a^d\delta_b^c) + \frac{1}{8}\Omega_{ab}\Omega^{cd} \tag{5.11}$$

that is an $USp(8)$ analog of $VV^{-1} = I$. The properties of the $\Gamma$-matrices, in particular, (5.8) imply the further relations:

$$(V^{-1})_{ab\ kl} V^{ij\ ab} = \frac{1}{2}(\delta_k^i\delta_l^j - \delta_k^j\delta_l^i), \quad (V^{-1})_{ab}^{i\alpha} V_{j\beta}{}^{ab} = \delta_i^j\delta_\alpha^\beta$$

and also

$$(V^{-1})_{ab}^{i\alpha} V_{kl}^{ab} = (V^{-1})_{ab\ kl} V^{\ i\alpha\ ab} = 0.$$

In the $SL(6,\mathbf{R}) \times SL(2,\mathbf{R})$ basis the element $P_{\mu\ ab}{}^{cd}$ is given by

$$\begin{aligned} P_{\mu\ ab}{}^{cd} &= (V^{-1})_{ij}^{cd} \nabla_\mu V_{ab}^{ij} + (V^{-1})^{cd\ i\alpha} \nabla_\mu V_{i\alpha\ ab} \\ &+ 2Q_{\mu[a}^{[c}\delta_{b]}^{d]} + gA_{\mu i}{}^j \left(2V_{ab}^{ik}(V^{-1})_{jk}^{cd} - (V^{-1})^{cd\ i\alpha} V_{j\alpha\ ab}\right). \end{aligned} \tag{5.12}$$

If we now require that $P_{\mu\ ab}{}^{cd}$ is in the coset space $E_6/USp(8)$, then the trace $P_{\mu\ ab}{}^{cb}$ should be equal to zero. This allows one to solve $Q_{\mu[a}^{[c}\delta_{b]}^{d]}$ via the vielbein:

$$Q_{\mu a}{}^b = -\frac{1}{3}\left((V^{-1})^{bc\ AB} \nabla_\mu V_{AB\ ac} + gA_{\mu i}{}^j (2V_{ac}^{ik}(V^{-1})_{jk}{}^{bc} - V_{j\alpha\ ac}(V^{-1})^{bc\ i\alpha}\right) \tag{5.13}$$

Substitution of the explicit expressions (3.2) yields

$$Q_{\mu a}{}^b = \frac{1}{24}\left(\Gamma_{in}\Gamma_{jn} - \Gamma_{i\alpha}\Gamma_{j\alpha}\right)_a{}^b \left(\nabla_\mu S S^{-1} + gSA_\mu S^{-1}\right)_i{}^j, \tag{5.14}$$

where on the r.h.s. the expression for the matrix $R_\mu$ defined by (3.3) appeared.

It is useful to note the following summation formula for $\Gamma$-matrices:

$$\Gamma_{in}\Gamma_{jn} - \Gamma_{i\alpha}\Gamma_{j\alpha} = -6\Gamma_{ij} - 7\delta_{ij} \cdot I.$$

Upon substituting this in (5.14), the term with $\delta_{ij}$ vanishes due to the tracelessness of $R_\mu$. Thus, we finally get

$$Q_{\mu a}{}^b = \frac{1}{4}(\Gamma_{ij})_a{}^b R_{\mu i}{}^j. \tag{5.15}$$

It is easy to see that $Q_{\mu a}{}^{b}$ is an antihermitian matrix indeed being an element of $Usp(8)$ Lie algebra, i.e., obeying the condition

$$Q_{\mu a}{}^{b} = -\Omega^{bc} Q_{\mu c}{}^{d} \Omega_{da}.$$

For the element $P_{\mu ab}{}^{cd}$ we, therefore, get

$$P_{\mu ab}{}^{cd} = \frac{1}{8}\left( (\Gamma_{in})^{cd} (\Gamma_{jn})_{ab} - (\Gamma_{i\alpha})^{cd} (\Gamma_{j\alpha})_{ab} \right) R_{\mu i}{}^{j} + 2 Q_{\mu [a}{}^{[c} \delta^{d]}_{b]}. \tag{5.16}$$

Since tensor $P_{\mu ab}{}^{cd}$ is completely fixed by the condition of the vanishing trace one now can check that (5.16) is indeed an element orthogonal to $USp(8)$-part of the Lie algebra of $E_{6(6)}$ w.r.t. to the Killing metric. Orthogonality means the following relation

$$P_{\mu ab}{}^{cd} U_{cd}{}^{ab} = 0, \tag{5.17}$$

where $U_{cd}{}^{ab} = Q_{\mu [c}{}^{[a} \delta^{b]}_{d]}$ is an element of the $USp(8)$ Lie algebra. Formula (5.17) then easily follows from (5.8), (5.9) and the relation

$$2 Q_{\mu [a}{}^{[c} \delta^{d]}_{b]} Q_{\mu [c}{}^{[a} \delta^{b]}_{d]} = 3 Q_{\mu a}{}^{c} Q_{\mu c}{}^{a} = -\frac{3}{2} (R_{\mu i}{}^{j} R_{\mu i}{}^{j} - R_{\mu i}{}^{j} R_{\mu j}{}^{i}).$$

We also need $(P_\mu)^{ab}{}_{cd} = \Omega^{aa'} \Omega^{bb'} \Omega_{cc'} \Omega_{dd'} P_{\mu a' b'}{}^{c'd'}$:

$$(P_\mu)^{ab}{}_{cd} = \frac{1}{8}\left( (\Gamma_{in})_{cd} (\Gamma_{jn})^{ab} - (\Gamma_{i\alpha})_{cd} (\Gamma_{j\alpha})^{ab} \right) R_{\mu j}{}^{i} - 2 Q_{\mu [c}{}^{[a} \delta^{b]}_{d]}.$$

Now we are ready to compute the scalar kinetic part of Lagrangian (3.1). By using the orthogonality condition (5.17) we can write it in the form

$$\begin{aligned} P_{\mu abcd} P_{\mu}^{abcd} = & \left( \frac{1}{8}\left( (\Gamma_{in})^{cd} (\Gamma_{jn})_{ab} - (\Gamma_{i\alpha})^{cd} (\Gamma_{j\alpha})_{ab} \right) R_{\mu j}{}^{i} + 2 Q_{\mu [a}{}^{[c} \delta^{d]}_{b]} \right) \\ & \times \left( \frac{1}{8}\left( (\Gamma_{km})_{cd} (\Gamma_{lm})^{ab} - (\Gamma_{k\beta})_{cd} (\Gamma_{l\beta})^{ab} \right) R_{\mu l}{}^{k} \right). \end{aligned}$$

After some algebra we arrive at the answer

$$P_{\mu abcd} P_{\mu}^{abcd} = 3 R_{\mu i}{}^{j} (R^{\mu})_{i}{}^{j} + 3 R_{\mu i}{}^{j} (R^{\mu})_{j}{}^{i} = \frac{3}{2} \operatorname{tr} \left( R_\mu + R_\mu^t \right)^2 . \tag{5.18}$$

Note that the r.h.s. of the scalar kinetic term appears to be manifestly positive in an Euclidean signature space as it should be.

# 6 Appendix B

### z-integrals

$z$-integrals are computed by using the technique by [17]. We list here the corresponding results:

$$\int \frac{d^5z}{z_0^5} G_\Delta(u) s^{I_3}(z) s^{I_4}(z) = \frac{1}{2^4\pi^4} \int d^4x_3 d^4x_4 \frac{s^{I_3}(\vec{x}_3) s^{I_4}(\vec{x}_4)}{x_{34}^2} K_1(w,\vec{x}_3) K_1(w,\vec{x}_4),$$

$$\int \frac{d^5z}{z_0^5} G_{\mu\nu}(u) s^{I_3} \overleftrightarrow{\nabla}^{\nu} s^{I_4}(z) = \frac{1}{2^4\pi^4} \int d^4x_3 d^4x_4 \frac{s^{I_3}(\vec{x}_3) s^{I_4}(\vec{x}_4)}{x_{34}^2} K_1(w,\vec{x}_3) \overleftrightarrow{\nabla}_{\mu} K_1(w,\vec{x}_4),$$

$$\int \frac{d^5z}{z_0^5} G_{\mu\nu\ \rho\lambda}(u) T^{\rho\lambda}(z) = \frac{1}{2^4\pi^4} \int d^4x_3 d^4x_4 \frac{s^{I}(\vec{x}_3) s^{I}(\vec{x}_4)}{x_{34}^2}$$

$$\left( g_{\mu\mu'} g_{\nu\nu'} + g_{\mu\nu'} g_{\nu\mu'} - \frac{2}{3} g_{\mu\nu} g_{\mu'\nu'} \right) \nabla^{\mu'} K_1(w,\vec{x}_3) \nabla^{\nu'} K_1(w,\vec{x}_4).$$

Note that computing the last integral, we have used the gauge freedom in the definition of the graviton propagator to obtain the answer in the simplest covariant form.

### Two-point function of lowest weight CPOs

As was noted in [29], a correct way to compute a two-point correlation function of operators in the boundary CFT, which is compatible with the Ward identitites, consists of two steps. First one uses the prescription by [2] for posing the Dirichlet boundary problem on gravity fields. Then one computes the two-point function in the momentum space and transform it further to the $x$-space. Below we undertake this procedure to find the two-point function of the lowest weight CPOs.

For a scalar field of the AdS-mass $m^2 = -4$ with the conventionally normalized quadratic action, the solution of the Dirichlet boundary problem reads as

$$K(z,k) = \left(\frac{z_0}{\varepsilon}\right)^2 \frac{K_0(kz_0)}{K_0(k\varepsilon)}$$

with the Fourier transform defining the following bulk-to-boundary propagator

$$K(z,\vec{x}) = -\frac{1}{2\pi^2\varepsilon^2 \ln\varepsilon} \left(\frac{z_0}{z_0^2 + |\vec{x}|^2}\right)^2 = -\frac{1}{2\pi^2\varepsilon^2 \ln\varepsilon} K_2(z,\vec{x}). \tag{6.1}$$

For the two-point correlation function in the momentum space we then have [29]:

$$\langle O(\vec{k}) O(\vec{k}') \rangle = \varepsilon^{-3} \delta(\vec{k}+\vec{k}') \lim_{z_0\to\varepsilon} \partial_{z_0} \left( \left(\frac{z_0}{\varepsilon}\right)^2 \frac{K_0(kz_0)}{K_0(k\varepsilon)} \right) = \delta(\vec{k}+\vec{k}') \frac{k}{\varepsilon^3} \frac{K_1(k\varepsilon)}{K_0(k\varepsilon)}.$$

where a nonessential local term $1/\varepsilon^4$ was omitted and $k$ denotes $|\vec{k}|$. Decomposing the result in power series, one gets

$$\langle O(\vec{k})O(\vec{k}')\rangle = -\delta(\vec{k}+\vec{k}')\frac{k}{\varepsilon^3}\frac{\frac{1}{k\varepsilon}+\sum_{n=0}^{\infty}\frac{(k\varepsilon/2)^{2n+1}}{n!(n+1)!}\left(\ln\frac{k\varepsilon}{2}-\frac{1}{2}\psi(k+1)-\frac{1}{2}\psi(k+2)\right)}{-\ln\varepsilon-\ln k+\ln 2-\psi(k+1)+\varepsilon(...)}$$
$$= \delta(\vec{k}+\vec{k}')\frac{1}{\varepsilon^4\ln\varepsilon}\left(1-\frac{\ln k}{\ln\varepsilon}+\frac{k^2\varepsilon^2}{2}\ln k+...\right).$$

The most singular relevant term here is the second one, so modulo local terms one finds

$$\langle O(\vec{k})O(\vec{k}')\rangle = -\delta(\vec{k}+\vec{k}')\frac{1}{\varepsilon^4\ln^2\varepsilon}\ln k.$$

Performing the Fourier transform, we finally get

$$\langle O(\vec{x}_1)O(\vec{x}_2)\rangle = \frac{1}{2\pi^2\varepsilon^4\ln^2\varepsilon x_{12}^4}. \tag{6.2}$$

In order to have a finite 2-point function in the limit $\varepsilon\to 0$ one has to rescale the boundary operator as $O(\vec{x})\to -\frac{1}{\varepsilon^2\ln\varepsilon}O(\vec{x})$, so that

$$\langle O(\vec{x}_1)O(\vec{x}_2)\rangle = \frac{1}{2\pi^2 x_{12}^4}. \tag{6.3}$$

To preserve the scale-invariance of the interaction term $\int d^4x O(\vec{x})s(\vec{x})$, where $s(\vec{x})$ is the boundary value of the bulk supergravity scalar $s(z)$ we then need to rescale the $s(\vec{x})$ in a way $s(\vec{x})\to -\varepsilon^2\ln\varepsilon s(\vec{x})$. After this rescaling the solution of the Dirichlet boundary problem reads as (4.8).

**Some identitites for $D$ functions**

As soon as $z$-integrals are performed, one is left with contact diagrams involving different numbers of derivatives. By using the identity [14]

$$\nabla_\mu K_{\Delta_1}(w,\vec{x}_1)\nabla^\mu K_{\Delta_2}(w,\vec{x}_2) = \Delta_1\Delta_2\left(K_{\Delta_1}(w,\vec{x}_1)K_{\Delta_2}(w,\vec{x}_2)-2x_{12}^2K_{\Delta_1+1}(w,\vec{x}_1)K_{\Delta_2+1}(w,\vec{x}_2)\right)$$

all the contact diagrams are then reduced to the sum of different $D$-functions.

In [16] some identities involving different $D$-functions were proved. We made use of the following ones:

$$x_{24}^2D_{2312}+x_{23}^2D_{2321} = D_{2211}-2x_{12}^2D_{3311},$$
$$2x_{12}^2D_{3311} = \frac{1}{2}x_{34}^2D_{2222}+\frac{1}{2}D_{2211},$$
$$x_{24}^2D_{1212} = x_{13}^2D_{2121}, \quad x_{14}^2D_{2112} = x_{23}^2D_{1221}$$
$$x_{13}^2x_{12}^2D_{3221}+x_{24}^2x_{34}^2D_{1223} = -\frac{1}{2}(x_{12}^2x_{34}^2+x_{13}^2x_{23}^2)D_{2222}$$
$$-\frac{3}{2}x_{14}^2D_{2112}+2x_{14}^4D_{3113}+\frac{1}{2}B.$$

and identities obtained from these by different permutations of indices to reduce the number of possible $D$-functions appearing in the 4-point function of the lowest weight CPOs to the minimal set giving by $D_{1212}$, $D_{2233}$ (with different permutations of indices) and $D_{2222}$. Here $B$ is a generating function for $D_{\Delta_1\Delta_2\Delta_3\Delta_4}$ and it is given by

$$B = \frac{\pi^2}{2}\int \frac{\prod d\alpha_j \delta(\sum \alpha_j - 1)}{(\sum \alpha_k \alpha_l x_{kl}^2)^2}.$$

**ACKNOWLEDGMENT**

G.A. is grateful to S. Theisen, S. Kuzenko and to A. Petkou, and S.F. is grateful to A. Tseytlin and S. Mathur for valuable discussions. The work of G.A. was supported by the Alexander von Humboldt Foundation and in part by the RFBI grant N99-01-00166, and the work of S.F. was supported by the U.S. Department of Energy under grant No. DE-FG02-96ER40967 and in part by RFBI grant N99-01-00190.

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